Linear Programming :-

We have following points :

(i) Set
(ii) Cartesian product of two sets
(iii) ordered pair (iv) linear expression
(v) linear equation (vi) Linear inequation/Inequalities
(vii) Constraint (viii) System of linear inequations
(ix) Solution set of linear equation
(x) Solution set of Constraints.
(xi) Consistent (xii) Inconsistent
(xiii) Graphical representation of linear equation, linear inequalities and solution set.

Set : Set is a collection of well defined and distinguished objects. Ex: $S = \{1, 2, 3, 4, 5\}$

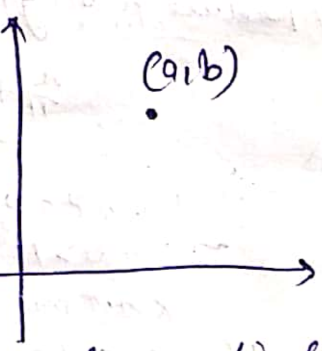
Cartesian product of two sets : Let A and B are be two sets; then $A \times B = \{(a,b) = \{\{a\}, \{a,b\}\} \mid a \in A, b \in B\}$

Ordered pair : The ordered pair (a,b) is the set of two elements $\{a\}$ and then set $\{a,b\}$. In general, $(a,b) \neq (b,a)$; if $a \neq b$.

$\Rightarrow \{\{a\}, \{a,b\}\} \neq \{\{b\}, \{b,a\}\}$

If $(a,b) = (c,d) \Leftrightarrow a = c$ & $b = d$.

Examples : The plane of coordinate Geometry is the set as ordered pair (a,b) with $a, b \in R$, where R is the set of real numbers.



& also a is called abscissa & b is called as ordinate.
or a is 1st component & b is second component of (a,b).

Linear expression:- The expression $ax+by+c$; a, b both are not zero simultaneously i.e. $a, b \in R$, then expression $ax+by+c$ is called linear expression over R in x & y.

e.g. $2x+y+1$; $3x+1$; y are all linear expression

but $0 \cdot x + 0 \cdot y + 5 = 5$ i.e. 5 is not linear expression.

Linear equation:- The expression $ax+by+c=0$; is called as linear equation; a, b both are not simultaneously zero. Ex. $2x+y=0$; $x-y+5=0$ are all linear equation.

Linear inequation / inequality:- The expression of the type $ax+by+c>0$; $ax+by+c \geq 0$ or $ax+by+c<0$, $ax+by+c \leq 0$; where a, b are not simultaneously zero ; is called as linear inequation.

e.g. $2x+y>0$; $2x+y+1 \geq 0$; $2x+y+1<0$; $2x-y \leq 0$ (are) all are inequations or linear inequalities.

Solution of linear equation:- Let $2x+y+1=0$ is a linear eqn ; $y=-1-2x$;

Taking $x=1$; $y=-1-2\times1=-3$;

$\Rightarrow x=1, y=-3$ is a soln.

Solution of linear inequation:- Let $2x+y+1>0$;

$y > -2x-1$

Take $x=1$; $y > -2\times1-1=-3$; $y>-3$

Take $y=4$; soln $x=1$; $y=4$.

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Constraints: A collection of linear equation or inequation or both is called a constraint provided it subject to some given condition.

Ex: Max $Z = 2x+3y$ —— (I)

s.t.

$$\left.\begin{aligned} x+y &> 0 \\ x &> 0 \\ y &> 0 \end{aligned}\right\} \text{—— (II)}$$

eqn (II) is called constraint.

System of linear inequations: — The collection of linear inequation is called system of linear inequation. e.g.

$$x+y>0$$
$$x>0$$
$$y>0$$

is a system of linear inequation.

Solution set of linear inequations: — Let $x \geq 0$ is a linear inequation, then 0, 1, 2, 3, —— all are solns but -1 is not soln.

So Solution set $S = \{0, 1, 2, 3, \ldots\}$ over integers.

Solution set of system linear inequations: — Let

$x+y>0$; $x>0$, $y>0$ over integers,

then $x=1, y=1$; $x=2\ y=1$; $x=1, y=2$, etc — are all soln.

Solution set $S = \{(a,b) \mid a, b \in N\}$.

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Consistent: The system of linear ineqn must have a soln. Then system of linear inequation is called consistent. As in previous example.

Inconsistent: If the soln set of system of linear inequation is empty, then system of linear inequation is called inconsistent. Ex.

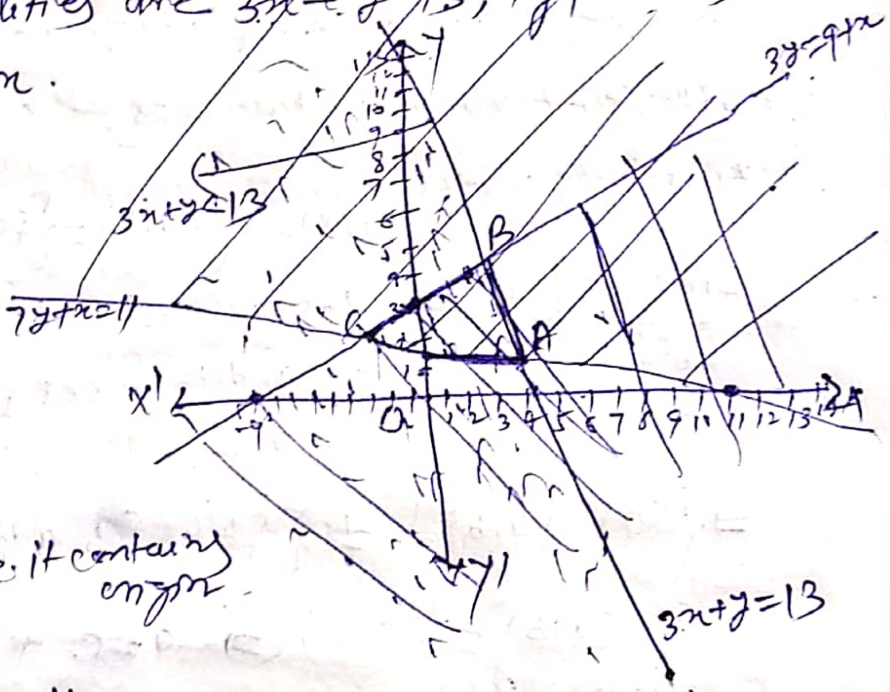
$x+y>0$; $x+y<0$; $x\geq 0$, $y\geq 0$.

Since there soln set is empty; so system of linear inequation is inconsistent.

Graphical Representation of Solution set: Let

Given inequalities are $3x+y\leq 13$; $7y+x>11$;

$3y\leq 9+x$.



$3x+y=13$

$\Rightarrow \frac{3x}{13}+\frac{y}{13}=1$

$\Rightarrow \frac{x}{\left(\frac{13}{3}\right)}+\frac{y}{13}=1$

Let $x=0$, $y=0$

$3\cdot 0+y<13$

true i.e. it contains origin.

$7y+x>11$

Reduce into $7y+x=11$

$\frac{x}{11}+\frac{y}{\left(\frac{11}{7}\right)}=1$

$x=0$, $y=0$

$7\cdot 0+0>11$

is not true.

It does not contain origin.

$3y\leq 9+x$

$3y-x\leq 9 \Rightarrow \frac{x}{(-9)}+\frac{y}{3}\leq 1$

$x=0$; $y=0$

$0<9$ is true;

so it containing origin

The area ABC in graph represents a soln set.

Example 1 :- Find the solution set of $x-1=0$ and $x<0$.

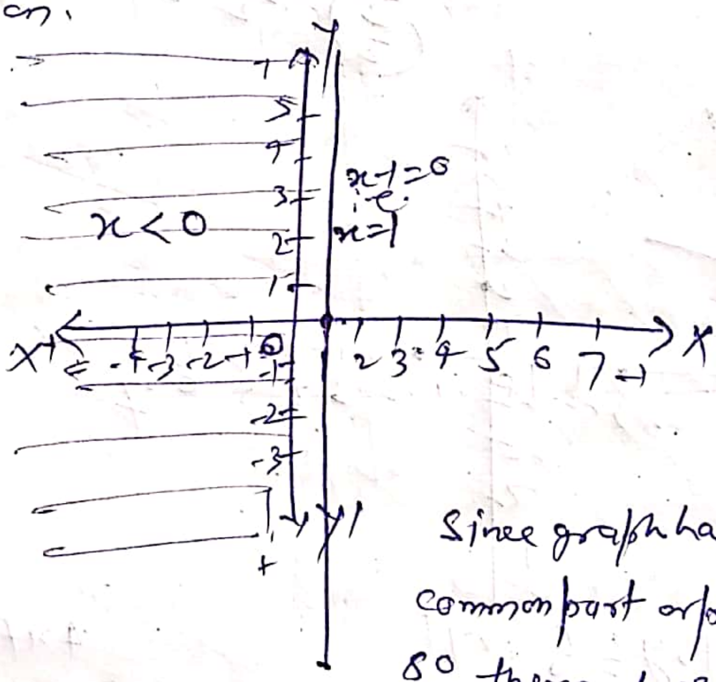
Solution :- Given that $x-1=0$ & $x<0$ —— (1)

$\Rightarrow x=1$; $x<0$

$\Rightarrow x=1>0$; $x<0$

Solution set $= \{ \} = \phi$

System of eqn or system of inequations or inequalities is inconsistent. i.e. has no solution.



Since graph has no common part or point; so there does not exist any soln.

Hence system of linear inequation and equations is inconsistent.

Example-(2). Find the solution set of $x+2y+1=0$ and $2x+4y+3=0$.

Solution :- We have $2x+4y+3=0$ —— (A)

$x+2y+1=0$ —— (B)

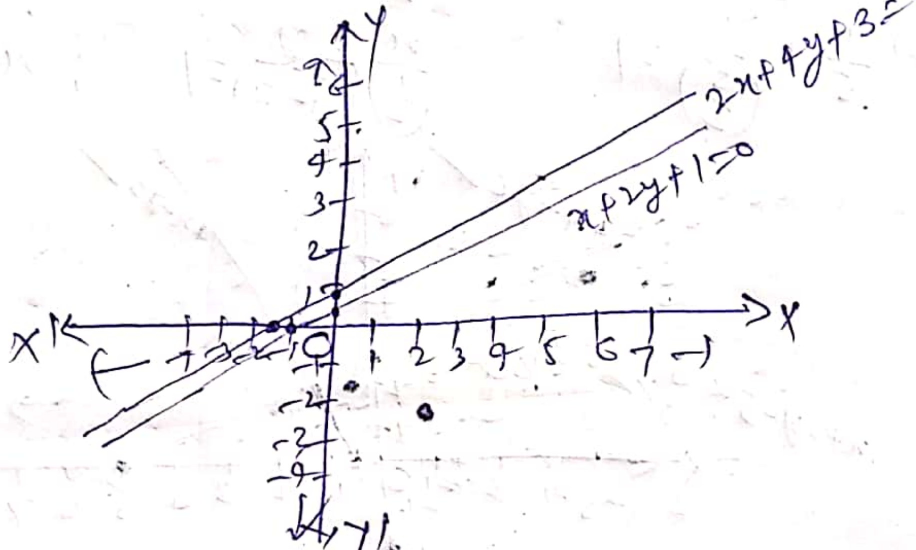
} —— (1)

multiplying eqn (1)(B) by 2 and subtracting from (1)(A) we have

$$2x+4y+3=0$$
$$\underline{-}2x\underline{-}4y\underline{-}2=0$$

$$\Rightarrow 1=0$$

This is absurd result.
means has no soln. system of eqn is inconsistent.



$2x+4y+3=0.$ $\qquad\qquad$ $x+2y+1=0$

$\Rightarrow -2x-4y=3$ $\qquad\qquad$ $-x-2y=1$

$\Rightarrow \frac{x}{\left(-\frac{3}{2}\right)}+\frac{y}{\left(-\frac{3}{4}\right)}=1$ $\qquad\qquad$ $\frac{x}{(-1)}+\frac{y}{\left(-\frac{1}{2}\right)}=1$

By graph; it clear that both lines are parallel mean its never intersect to each other, Hence no point is common. So there are no solution. Therefore system of eqn is inconsistent.

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Example ③ :- Find the graph of $x+2y-5<0$, $4x-y<2$ and $y>0$. On the graph mark three points by black dot.

Solution :- Given that

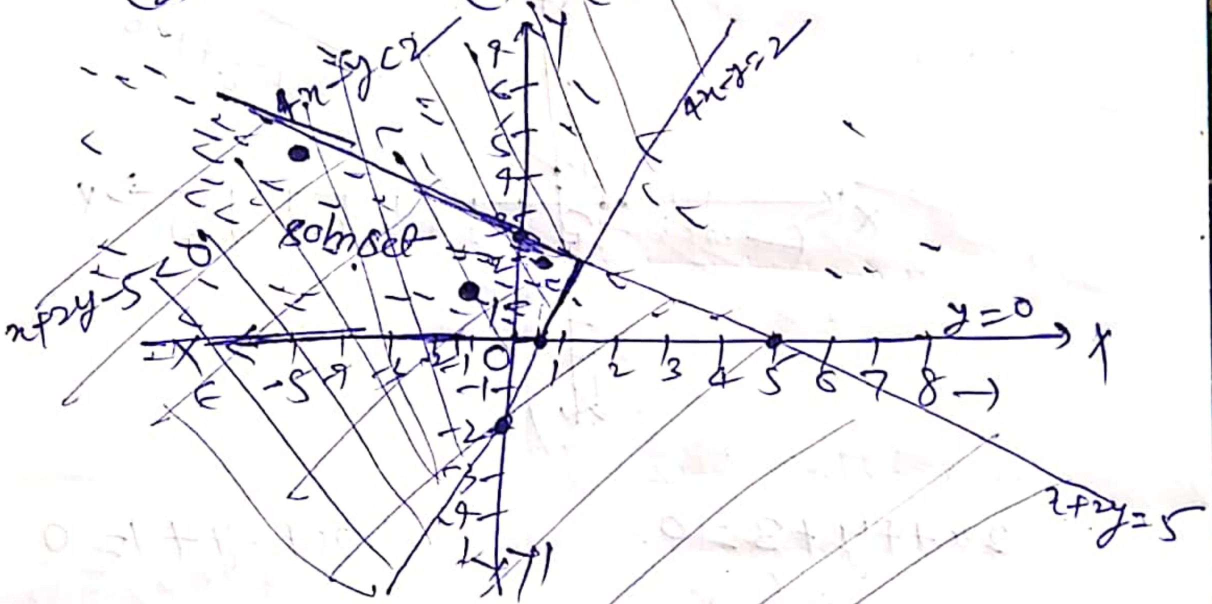
$$x+2y-5<0 \;,\; 4x-y<2 \;;\; y \geq 0 \quad \text{—(i)}$$

reduces into eqn; we have

$$x+2y-5=0 \qquad 4x-y=2 \;;\; y=0 \quad \text{—(ii)}$$

$$x+2y=5 \qquad \frac{4x}{2}-\frac{y}{2}=1 \;;\; y=0$$

$$\Rightarrow \frac{x}{5}+\frac{y}{\left(\frac{5}{2}\right)}=1 \qquad \frac{x}{\left(\frac{1}{2}\right)}+\frac{y}{(-2)}=1 \;;\; x\text{-axis.}$$



Putting $x=0$; $y=0$

$0+2\cdot 0-5<0$

$\Rightarrow -5<0$

True.

soln set containing origin

$0\cdot x-0<2$

$\Rightarrow 0<2$

True

soln set containing origin.

$y>0$

Above of x-axis.

Common part in graph gives solution set of system of linear inequations.

$\left(\frac{1}{2},2\right)$; $(-1,1)$, $(-5,4)$ are the three points.

But we comment that there in infinite number of soln exist.

Example 4: Draw the graph of $4x+3y \le 6$. Mark two solutions of this on the graph.

Solution: We have

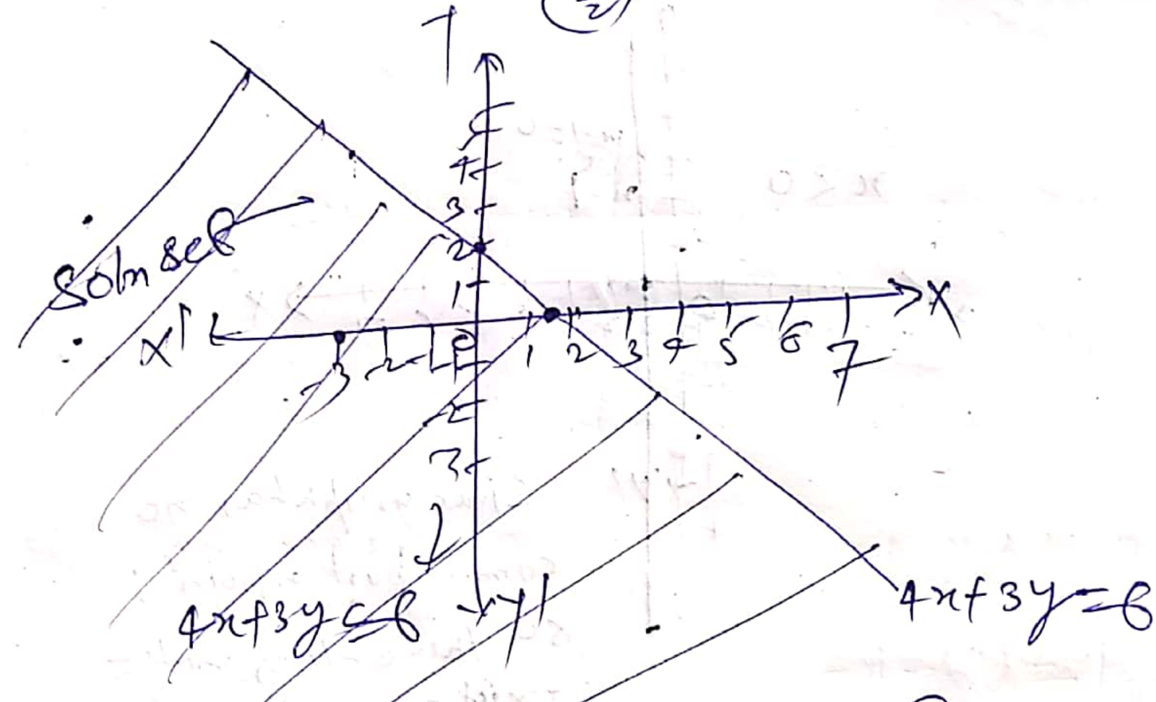
$4x+3y \le 6$ —— (I)

Reduces into

$4x+3y = 6$ —— (II)

$\Rightarrow \frac{4x}{6} + \frac{3y}{6} = 1$

$\Rightarrow \frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{2} = 1$



Putting $x=0$; $y=0$ in (I)

$4\cdot 0 + 3\cdot 0 \le 6 \Rightarrow 0 \le 6$ True

So soln set containing origin.

say $(-3,6)$, $(-6,2)$

Another point $(\frac{3}{2}, 0)$; $(0,2)$ also on graph

Introduction of operation Research (O.R.) :-

The term operation research, was first coined in 1940 by Mc Closky and trefthen in a small town, Bowdsey of united kingdom. The new science came into existence in military context during world war II. Military Management called on scientists from various disciplines, an organised them into term to assist in solving strategic and tactical problem, i.e. to discuss, evolve and suggest ways and means to improve the execution of various military projects. By their join efforts, experience and deliberations, they suggested certain approaches that showed remarkable progrees. This new approach to systematic and scientific study of the operation of the system was called the operation Research or Operational Research (O.R.).

The operation research society of America was formed in 1950. Other countries followed suit, and in 1957 the International Federation of Operation Research societies was established.

Definition of operation Research :- There are a number of definitions to understand the nature of O.R.; we have two only, Here, (i) In the words of T.L. Satty, "O.R. is the art of giving bad answers to problems where otherwise worse answers are given."

(ii) According to H.A. Taha, "O.R. is a scientific knowledge through inter disciplinary team effort for the purpose of determining the best utilization of limited resources."

Objective of Operation Research (O.R.):

In short the objective of O.R. is to provide a scientific basis to the decision makers for solving the problem involving the interaction of various components of the organization by employing a team of scientists from different disciplines, all working together for finding a solution which is in the best interest of organization as a whole. The best solution thus obtained is known as optimal decisions.

Application of Operation Research:

Operation Research is mainly concerned with the techniques of applying scientific knowledge, besides the development of science. It provides an understanding which gives the expert / manager new insights and capabilities to determine better solutions in his decision making problems, with great speed, competence and confidence. In recent years, O.R. has successfully entered many different areas of research in defence, Government, Service, organisation & Industry. We shortly describe some applications of O.R. in the functional areas of management:

(i) Finance, Budgeting and Investment (ii) Marketing
(iii) Physical Distribution (iv) Personnel
(v) Purchasing Procurement and Exploration.
(vi) Productions. Besides the mentioned applications of O.R. in the context of modern management, its use has now extended to a wide range of problems, such as the problems of communication & informations, socio-economics fields & national planning &

Introduction of Linear Programming (Problem):—

Linear Programming (L.P.) is a mathematical technique for the analysis of optimum decisions subject to certain constraints in the form of linear inequalities.

Mathematically speaking, it applies to those problems which requires the solution of maximization or minimization problems subject to a system of linear inequalities stated in terms of certain variables.

The term "linear" indicates that the function to be maximized is of degree one and the corresponding constraints are represented by a system of linear inequalities. The word "programming" means that the planning of activities in a manner that leads to some optimum results with limited resources. A programme is optimal if it maximizes or minimizes output, profits or cost of a firm.

Linear programming may thus be defined as a method to decide the optimum combination of factors (inputs) to produce a given output or the optimum combination of products (outputs) to be produced by the given plant and equipment (inputs). It is also used by a firm to decide between varieties of techniques to produce a commodity.

Limitations of Linear programming:—

Linear programming has turned out to be a highly useful tool of analysis for the business executives. It is

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